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PRACTICAL DESIGN OF SOLID-BARREL, REINFORCED-CONCRETE SKEW STRUCTURES

By Bernard L. Weiner

STRUCTURAL DIVISION

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PAPERS

PRACTICAL DESIGN OF SOLID-BARREL, REINFORCED-CONCRETE SKEW STRUCTURES

By Bernard L. Weiner,[1] M. ASCE

Synopsis

No new theories are presented in this paper. Its purpose is rather to show the simplifications that are possible in the application of existing theory. These simplifications have been tested in practice. A solid-barrel skew frame can be designed with little more work than that required for the design of a similar right-angle frame. Theoretically, it is true that a skew frame must be analyzed in three dimensions, whereas a right-angle structure can be analyzed in a single plane. It is to be expected, therefore, that there will be stresses in the former type of frame which either do not exist in the latter type or exist only to a very moderate extent. Although these additional stresses (often referred to as "skew stresses") are important, provision can be made for them, for practical design purposes, with very little additional work.

Introduction

It is the purpose of this paper to show that by taking advantage of certain characteristics of skew frames and of certain other factors that follow directly from considerations of static equilibrium, permissible approximations in existing theories may be made which reduce the labor involved in the design of a skew frame to little more than that required for a similar right-angle structure.

That these approximations are possible should not be very surprising. It happens often that "practical design" differs considerably from "theoretical analysis" for research purposes. Where "accurate" formulas are too complex and cumbersome for everyday use, approximate methods are developed by or for the practicing engineer; in other words, "engineering design" often becomes merely "conventional design."

Note.—Written comments are invited for publication; the last discussion should be submitted by April 1, 1951.

[1] New York, N. Y.

For "small" skews, the expedient of designing on the skew span has often been used. A consideration of the arguments in this paper will show that this expedient is entirely incorrect; it leads to results that either are on the side of danger or are very wasteful. The former is true in the case of members which are essentially horizontal; the latter is true for members which are essentially vertical. Only in the case of a member that is essentially horizontal and that is subjected to uniformly distributed vertical loads are the results correct; for the same loading, the results are wasteful for essentially vertical members. With the simplifications described in this paper, this expedient, besides being incorrect, also becomes unnecessary.

The methods described have been used since about 1930 for the design of numerous structures (such as those on the Westchester County Parkway System in New York and on the Belt Parkway System in New York, N. Y.) with skew angles as great as 50°, with soffits of various shapes, and of one, two, and three spans. The thickness of the members and the amount of reinforcing steel do not appear excessive or unusual compared to that required for a right-angle structure of span equal approximately to the skew span. Such skew structures have been built and have been in service from 6 years to 20 years without showing signs of distress.

In recent years, a few splayed bridges, with faces not parallel (such as those designed for the Cross-Bronx Expressway in New York City) and bridges with curved faces (such as those on the Saw Mill River Parkway in Westchester County and on route 4 Parkway in New Jersey), have been built. An extension of the principles presented in this paper leads to a practical procedure for the design of this type of structure also.

Characteristics of Skew Frames

The practical design of most structures—as distinct from the theoretical analysis for research purposes—may be divided into two steps:

1. The computation of the total moments, thrusts, and shears; and
2. The design of the sections to resist the total stresses computed in step 1.

It should be remembered that, in a very real sense, the total stresses—especially the moments—are mere mathematical abstractions; that is, the effects of the loads on a structure are resisted by unit stresses and not by moments and total forces. In some types of structures such as narrow beams, it is possible to express the sum, or resultants, of the unit stresses in terms of couples, about some more or less arbitrary axis, and in terms of total forces. In the case of wide structures such as slabs, total stresses have little significance and are practically useless for design purposes. In the latter case, it is stated that the distribution must be known—which merely begs the question.

As a matter of fact, in the accurate theory of slabs, the unit stresses are first found from the elastic equilibrium and elastic deformation equations of the mathematical theory of elasticity, stated in terms of partial differential equations—and then, working backward, the moments and shears and their so-called "distribution," are found. The meaning of "distribution," in this case, is that the unit stresses can be summed up in one direction (the depth of the

slab), resulting in moments and shears per unit length in the other two directions. An analogous situation exists in the case of the accurate theory for the bending of narrow beams, although, in this instance, summing up over the depth gives more complete information and the resultant moments and the total thrusts and shears have more real significance.

Fortunately, since the "run-of-the-mine" structure is of the narrow beam type, it is possible, in most cases of importance, to sum up the unit stresses in terms of total moments and forces—"packaged" quite neatly for the purpose of discussion and design. Furthermore, in such cases it is also possible to compute the resultant stresses directly. In other words, engineers usually do designs backward. The method is convenient, however, where it can be used.

Failure to remember the real nature of everyday design procedure has often resulted in confusion, and this has been particularly true of the approach to the analysis of skew structures of the solid-barrel type.

The analysis and the design of a rectangular frame or arch, for example, are really based on the assumption that the frame or arch can be divided into narrow strips, each one of which may be designed as a narrow beam—because of symmetry along the width of the structure. Although it is not possible—in abstract theory—to follow this procedure for skew structures, a method for finding the total stresses, based partly on analogy, has nevertheless been developed. There is no doubt, however, as to the correctness of the method for finding the total stresses since the computed results have been checked by tests.

It must be remembered that the computation of the total stresses presupposes unit stresses of which the former are merely the resultants; that is, the designer must work backward to find the underlying unit stresses. The direct method, of course, would be to find the unit stresses from the equilibrium equations of the mathematical theory of elasticity, but the complexity of the problem makes the indirect method a necessity. Even this indirect method, in its full unabridged form, is rather complex, long, and tedious. Fortunately, extensive simplifications can be made for practical design purposes—so much so that the design of a skew structure need not require much more work than that required for a similar rectangular one.

In a narrow, beam type of structure or in a rectangular wide one treated as a series of narrow beams indentically loaded, the resultant forces on any section are a bending moment, a thrust, and a shear. Similar total stresses exist in a skew structure; but, in addition, there are also a torsion moment, another bending moment, and another shear, as shown in Fig. 1—that is, six total stresses altogether. Theoretically, the six stresses cannot be separated except in at least one special case where the median or "neutral" surface is straight, truly vertical, and truly horizontal. For this special case, the writer has shown, elsewhere,[2] that the stresses which exist in a rectangular frame and those which exist only in a skew frame (loaded symmetrically with respect to their widths) are separable and may therefore be treated as two separate

[2] "Design of a Reinforced Concrete Skew Arch," by Bernard L. Weiner, *Transactions*, ASCE, Vol. 96, 1932, p. 1281, part III.

elastic systems. Simplifications result from this consideration for, instead of having six unknowns for each span, there are now two sets of three unknowns.

It has been found in the general case, however, that, where the neutral surfaces of the several members are neither straight nor vertical and horizontal, there is little error in considering the two sets of stresses independently and in analyzing them as two separate elastic systems. The closeness of the approximation has been checked for a single-span symmetrical rigid frame by actual computation, and by comparison of the stresses obtained from the "exact" theory with those obtained from the approximate analyses.[3] Similar results were later obtained by Richard M. Hodges[4] and by others. The approximation holds good for any number of spans and for any shape of frame or arch that would be encountered in practical design.

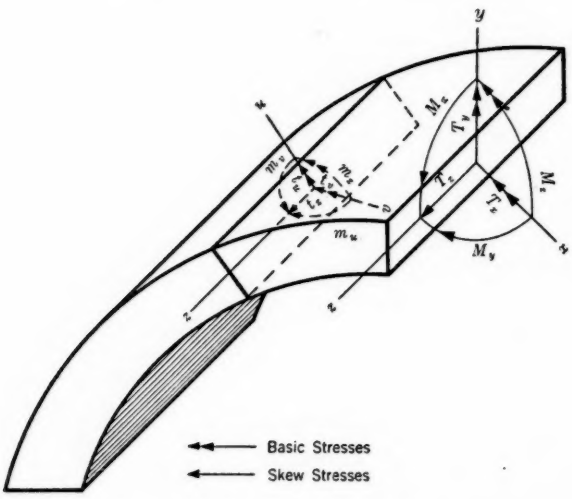

FIG. 1.—TOTAL STRESSES

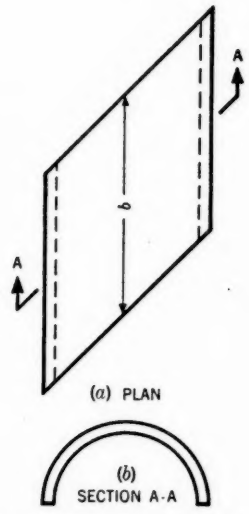

FIG. 2.—REPRESENTATIVE PLAN AND SECTION

One important result from this approximation is that, except for temperature and shrinkage, the set of three equations for those total stresses which exist also in a right frame takes the same form as in a right frame. In other words, these stresses—hereafter to be referred to as the "basic stresses"—are equal to those in a rectangular frame of the same square span and of the same elastic and geometric properties. Therefore, it is possible to obtain the basic stresses from a right-frame analysis, with exceptions to be noted.

The basic stresses for temperature and shrinkage present no great difficulty. Analysis shows that they are equal to those in a right frame multiplied by the square of the secant of the skew angle.[2] Although these approximations reduce the work considerably, study of the internal stress distribution leads to further simplification. It will be shown that, in most cases, the "skew stresses"

[3] "Design of a Reinforced Concrete Skew Arch," by Bernard L. Weiner, *Transactions*, ASCE, Vol. 96, 1932, p. 1286.

[4] "Simplified Analysis of Skewed Reinforced Concrete Frames and Arches," by Richard M. Hodges, *ibid.*, Vol. 109, 1944, p. 913.

need not be computed at all for design purposes. Designs that are adequate for both safety and economy can be made by applying a rather simple correction to a right-frame design.

The right-frame analysis for the total stresses can be made by any of the methods in common use and this phase of the problem, except for one detail, therefore, needs no further discussion. Theoretically, a skew frame cannot be divided into narrow strips and must therefore be treated in three dimensions. It is found, however, that the skew width b, measured parallel to the abutments, can be factored out of all the terms of the equations but one, the so-called "factor of torsion." This term is equal, however, to a constant multiplied by a factor $\frac{1}{b^3} + \frac{1}{t^3}$; and, since $\frac{1}{b^3}$ is small compared to $\frac{1}{t^3}$ (t being the total depth of any section), there is practically no loss of accuracy if the former is neglected even in the complete skew-frame analysis. Thus, the resulting stress computations made on this basis—with b factored out—give the total stresses divided by b, which may or may not be the stresses per unit width, depending on whether or not the stresses are uniformly distributed. In most cases, however, this point is merely of academic importance, as will be shown.

In theory, then, the skew frame is treated as a whole in three dimensions, and dividing by b is merely an arithmetical convenience. Nevertheless, it is possible to use a strip of unit width provided a proper "sample" is taken. Since in the unabridged theory, the square span is used, it must also be used in the strip. The proper section for the substitute right frame is illustrated in Fig. 2. Projecting a skew section on to a plane perpendicular to the abutments results in a representative "sample" for analyzing an equivalent right frame with the same square span as the skew frame and with the same elastic and geometric properties.

Stress Distribution and Boundary Conditions

Considerable information about the action of a structure may be obtained from a qualitative analysis of the boundary conditions. It is well known for instance that unit shears (normal to the same axis), on planes perpendicular to each other, must be equal—which leads to the important corollary that, at the intersection of a section with a free surface at right angles to it, the unit shears must be zero. The same reasoning leads to the more general corollary that, if the section and the free surface are not perpendicular to each other, the unit shears cannot be zero. Stated in another way, at the intersection of a plane with a free surface oblique to it, the shears must have such a value that the resultant of all stresses will be parallel to the free surface. This conclusion follows from the fact that any other resultant will have components that can be kept in equilibrium only by stresses on the free surface, which, of course, is impossible—or is a contradiction in terms. It follows, therefore, that at the boundary, if the direct stresses are known, the shears are also known—because the direction of the resultant is known.

Since the boundary values of the unit shears are independent of the total shears and torsion moments, and are dependent only on the direct stresses,

only the latter need to be computed for the design of the fascia sections. It is implicitly assumed in the unabridged theory for the computation of the total stresses that the ordinary bending moment and the direct thrust are uniformly distributed; and, since this theory has been checked by tests, the same assumption can logically be made for the design of the sections. This assumption has

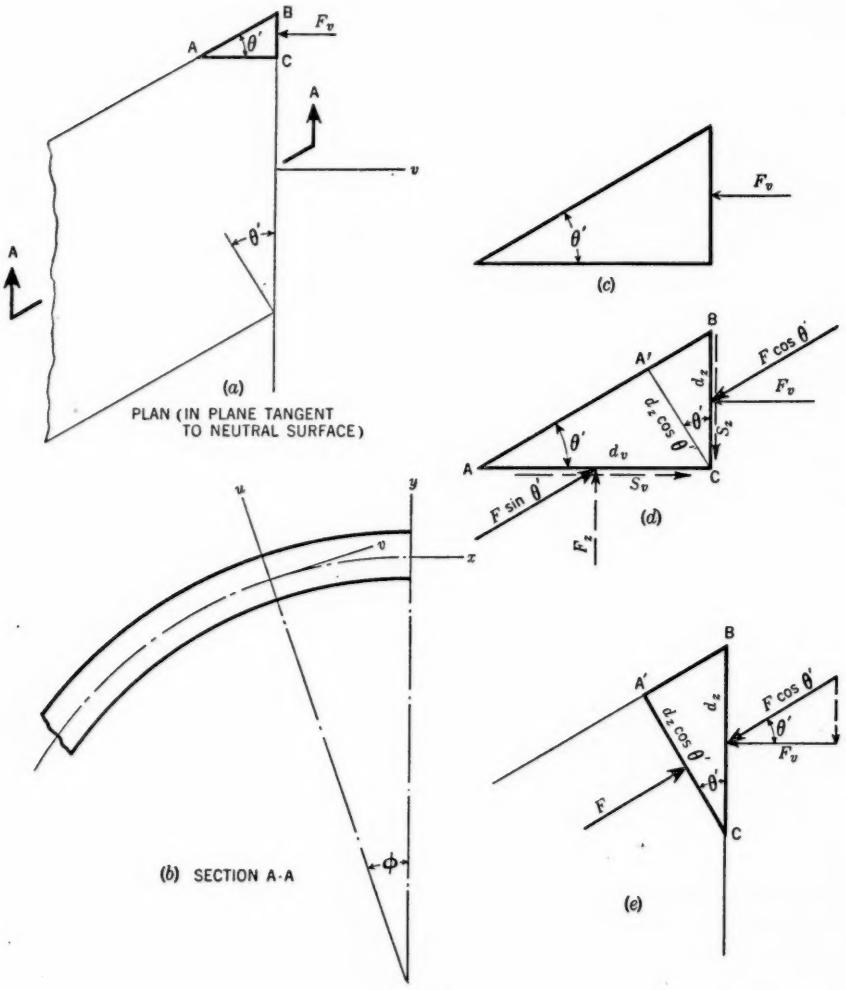

FIG. 3

been made in all skew-arch design theory (and also for right arches). Until a study of this point is made, based on the mathematical theory of elasticity and substantiated by tests, no exact answer can be obtained. The total basic stresses are nearly equal, however, to those in a rectangular frame of the same square span, as previously explained. Hence, it follows that, as far as the

fascia sections are concerned, only a right-frame analysis need be made to obtain the total stresses. Although it is true that the second bending moment, M_y at the crown (or m_u elsewhere), produces the maximum stresses at the fascia, the latter are so small that they are neglected even in an unabridged design.

Resultant Moment and Thrust per Unit Width at Fascia

The force shown in Fig. 3(*c*) is assumed to be known, as previously explained. For equilibrium, this force, F_v per unit width, must be accompanied by the other three forces per unit width shown as dashed-line vectors in Fig. 3(*d*). The value of each of these forces in terms of F_v and of the modified skew angle, θ' (the value of which in terms of the skew angle, θ, will be determined later) is easily found from considerations of equilibrium; thus, because $\Sigma V = 0$:

$$S_v\,dv = F_v\,dz \quad \ldots\ldots (1)$$

and

$$S_v = F_v \frac{dz}{dv} = F_v \tan\theta' = \epsilon' F_v \quad \ldots\ldots (2)$$

Since $\Sigma M = 0$, writing ϵ' for $\tan\theta'$, taking moments about point C, for convenience,

$$F_z \frac{\overline{dv^2}}{2} = F_v \frac{\overline{dz^2}}{2} \quad \ldots\ldots (3)$$

and

$$F_z = F_v \left(\frac{dz}{dv}\right)^2 = F_v \tan^2\theta' = (\epsilon')^2 F_v \quad \ldots\ldots (4)$$

For $\Sigma Z = 0$,

$$S_z\,dz = F_z\,dv \quad \ldots\ldots (5)$$

$$S_z \frac{dz}{dv} = F_z \quad \ldots\ldots (6)$$

$$\epsilon' S_z = (\epsilon')^2 F_v \quad \ldots\ldots (7)$$

and

$$S_z = \epsilon' F_v = S_v \quad \ldots\ldots (8)$$

Eq. 8 proves the well-known fundamental relationship that the unit shears (normal to the same axis) on planes perpendicular to each other are equal.

Also,

$$\frac{S_z}{F_v} = \frac{\epsilon' F_v}{F_v} = \epsilon' \quad \ldots\ldots (9)$$

The resultant, therefore, is parallel to the boundary (face of structure), as it should be. To find the resultant stress on surface BC:

$$F\,dz\cos\theta' = \sqrt{(F_v\,dz)^2 + (S_z\,dz)^2} = dz\sqrt{F^2_v + (\epsilon')^2 F^2_v} \quad \ldots\ldots (10)$$

$$F = F_v \sec^2\theta' \quad \ldots\ldots (11)$$

Also,

$$\frac{F_z}{S_v} = \frac{(\epsilon')^2 F_v}{\epsilon' F_v} = \epsilon' \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

The resultant on surface AC, Fig. 3(*d*), is therefore equal and opposite to that on line BC—as it should be, since surface AB is a free surface or boundary.

To find the resultant stress on surface AC, Fig. 3(*d*):

$$F\ dv \sin\theta' = \sqrt{(S_v\ dv)^2 + (F_z\ dv)^2} = dv\sqrt{S^2_v + F^2_z} \ldots\ldots\ldots (13)$$

$$F \sin\theta' = \sqrt{(\epsilon')^2 F^2_v + (\epsilon')^4 F^2_v} = \epsilon' \sec\theta'\ F_v \ldots\ldots\ldots\ldots (14)$$

and

$$F = F_v \sec^2\theta' \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

The foregoing derivation is necessary for an unabridged design and is repeated for completeness. For the present purpose, only Eq. 11 is necessary and it can be derived more directly.

In Fig. 3(*e*), for equilibrium (since surface A′B is a boundary and can have no stresses on it, and, since there can also be no horizontal shear on line A′C which is normal to it),

$$F\ dz \cos\theta' = F_v\ dz \sec\theta' \ldots\ldots\ldots\ldots\ldots\ldots (16)$$

and

$$F = F_v \sec^2\theta' \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

Attention is called to the fact that the quantity F_v can be either a moment or a thrust per unit width.

Stresses in the Interior Parts of a Section

The stresses in the interior parts of a section differ from those at the fascia and the question is whether safety and economy permit the use of the fascia design for the entire width of structure. If this is possible, then it becomes unnecessary to use the skew-arch analysis for the computation of the total stresses—that is, neither the "exact" nor the approximate skew-arch analysis is necessary for design purposes. Its use then becomes largely restricted to analysis for research purposes only. Elsewhere,[5] the writer has given typical examples of a complete design, leading to the conclusion that the design of the fascia, only, is sufficient. Later designs, for single-span and multiple-span frames and full centered "arches" (frames with curved soffits) and for skew angles up to 50°, also showed that adequate designs can be made without recourse to complete skew-arch analysis.

In the "accurate" design of the sections, the total stresses must first be reduced to stresses per unit width; and then, by a process exactly the same as that used in the ellipse of stress for unit stresses, the maximum unit bending moments on principal planes, as well as the maximum unit torsion moments, are found. Although the maximum unit thrusts and shears may also be found,

[5] "Design of a Reinforced Concrete Skew Arch," by Bernard L. Weiner, *Transactions*, ASCE, Vol. 96, 1932, pp. 1273–1281.

their computation usually is not necessary since the normal practice of determining maximum unit stresses from the moments is also applicable to this case.

Formulas for computing the various moments and thrusts per unit width (see Fig. 4), which were developed elsewhere,[6] are

$$t_s = \frac{3}{2}\left(\frac{t_z - \epsilon' t_v}{b}\right)\left(1 - \frac{4z^2}{b^2}\right) + \epsilon' \frac{t_v}{b} + 12\frac{\epsilon' m_u z}{b^3} \quad \ldots\ldots (18)$$

$$t'_n = \epsilon' t_s \quad \ldots\ldots (19)$$

At the face of bridge,

$$m_s = \frac{\epsilon' m_z}{b} \quad \ldots\ldots (20a)$$

and, at any other point of the section,

$$m_s = \frac{m_v + \epsilon' m_z}{2b} \quad \ldots\ldots (20b)$$

At the face of bridge,

$$m' = \frac{(\epsilon')^2 m_z}{b} \quad \ldots\ldots (21a)$$

and, at any point of the section,

$$m' = \frac{\epsilon' m_v}{b} \quad \ldots\ldots (21b)$$

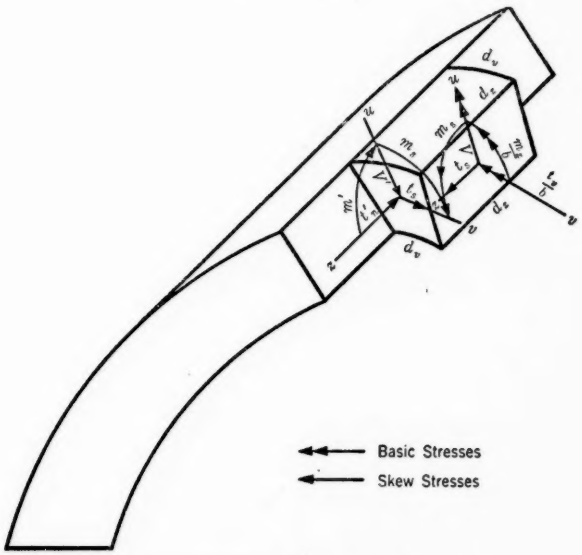

FIG. 4.—STRESSES PER UNIT WIDTH OF FRAME

At the boundary (face of bridge) only the basic stresses need be known, and all these are special cases of the generalized stress, F_v, in Eqs. 2, 4, 8, and 11. Equations for the radial shear (parallel to the u-axis or the y-axis) are not

[6] "Design of a Reinforced Concrete Skew Arch," by Bernard L. Weiner, *Transactions*, ASCE, Vol. 96, 1932, pp. 1251–1260 and p. 1297.

given as these shears are of such a magnitude that they do not govern the design in either a right or a skew solid-barrel frame.

All the formulas thus far developed are in terms of θ', the modified skew angle, which is the angle of skew in a plane tangent to the neutral surface at any section. The value of θ' at any section may be easily computed from the known skew angle, θ in the horizontal plane.

From Fig. 5 (writing $\epsilon = \tan\theta$ and $\epsilon' = \tan\theta$ for brevity), and

$$\epsilon' = \tan\theta' = \tan\theta\cos\phi \quad \ldots\ldots\ldots\ldots\ldots\ldots (22)$$

$$\tan\phi' = \tan\phi\cos\phi \quad \ldots\ldots\ldots\ldots\ldots\ldots (23)$$

The complete designs showed that for moderate skews (less than 35°) there is little difference at the crown between the fascia and the interior parts. Furthermore, at the haunches or springing lines of the frames there was practically no difference for all skew angles up to 50°, the stresses at the fascia always being slightly larger. For skew angles of from 35° to 50°, the crown stresses at the fascia were considerably larger than those in the interior, the ratio increasing with the higher skews. However, the loss in economy is not so great as would appear at first glance.

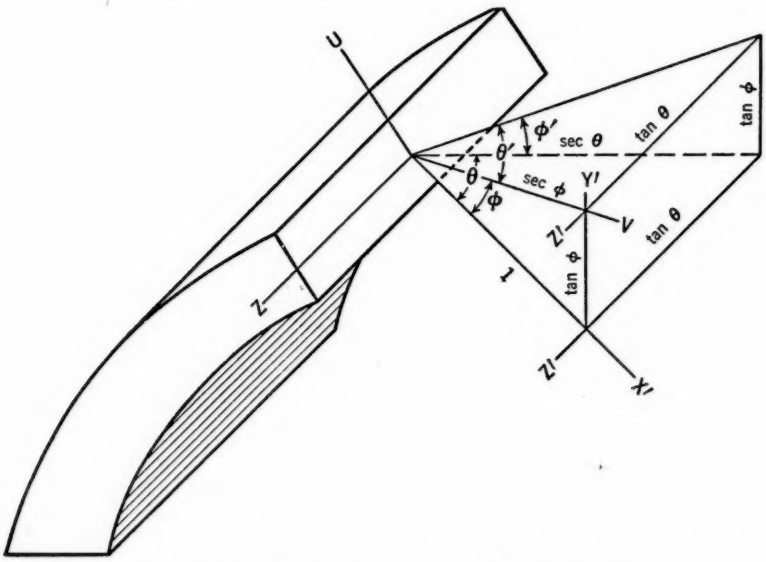

FIG. 5.—GEOMETRY SHOWING RELATIONSHIP BETWEEN θ AND θ'

Analysis shows that the fascia section extends for a distance of three or four times the depth of the section—and, for safety, the maximum depth should be used. Thus, for a haunch depth of 5 ft (which is very common and which increases with the skew) the two fascias account for 40 ft of the width. Also, the crown steel extends for only a short distance; hence, the saving in steel would be slight. There would be no saving in concrete as it usually would not be practical to vary the depth of the section.

Furthermore, sound engineering judgment would indicate that the maximum section of the fascia should be used for the entire width. Although the writer believes that the distribution formulas (Eqs. 18 to 21) are based on logical assumptions, no exact quantitative theory or tests are available. Such tests as have been made are qualitative in nature and rather crude and of little value, even though they do tend to indicate that, at the fascia, the stresses are parallel to the boundary.[7] From the nature of the problem, the stress distribution at the fascia is more certain; and, since the difference becomes appreciable only for the larger skews, it seems to the writer that caution would indicate the use of the larger fascia stresses. In any case, until a theory or accurate test data become available, refinements do not seem to be justified.

The so-called skew stresses (total) are of such a nature, that, even if it is considered necessary to compute them, sufficiently accurate results for design could be obtained from charts that could be easily prepared.

Analysis shows that horizontal shears of considerable magnitude exist in a skew frame; but, since they are more in the nature of column shears, they are of no serious consequence. It can be shown, in the case of a column that, on a plane at 45° with the axis, shears exist which are equal to one half of the compression stress, or as high as 400 lb per sq in. or 500 lb per sq in. The largest shear found was at the crown of a frame with a 50° skew; it varied from about 200 lb per sq in. in the compression area to 100 lb per sq in. (in the opposite direction) in the tension area. At the haunch, the shears were of the same magnitude as that permitted in a beam fully reinforced for tension. In both cases, these high shears were at the center of width, *b*.

It is recommended, therefore, that the larger the skew, the greater the relative thickness of concrete should be; that is, as the skew increases, less compression reinforcement should be used, thus resulting in thicker crown sections. Skews of much more than 50° become impractical for purely geometric reasons; and, for this upper limit, it seems wise to make the crown sufficiently deep so that it can be designed without any compression steel. Although the single-span frame with a 50° skew, previously mentioned, was designed with rather heavy compression reinforcement and showed no signs of distress after almost 20 years of service, the practice is not recommended—unless a complete design is made.

Splayed Bridges and Bridges with Curved Faces

The history of the skew frame or arch bridge is usually prefaced by the statement that this type of structure was made necessary by the development of the high-speed automobile. The trend of fitting the bridge to the highway rather than the reverse practice, which was so common even about 1925, has now become more marked. Since entrances and exits to and from an express highway are logically to be found at grade separations, bridges with nonparallel faces are becoming quite common. The practice of fitting a straight bridge to a curved highway is gradually being abandoned. The result has been that bridges of the shapes (in plan view) shown in Figs. 6(*b*), 6(*c*), and 6(*d*) are being designed and built fairly frequently.

[7] Discussion by B. L. Weiner of "An Analysis of Multiple-Skew Arches on Elastic Piers," by J. Charles Rathbun, *Transactions*, ASCE, Vol. 98, 1933, p. 46.

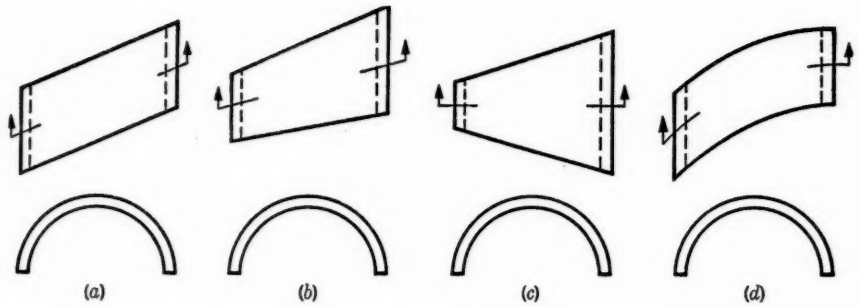

FIG. 6

There is nothing implicit in the unabridged skew-arch theory which prevents its application to any of the structures with parallel abutments shown in Fig. 6. A modification of the definition of the "skew angle" (and the consequent trigonometric changes) is all that is necessary. With the simplified design already described, the representative "sample" frame for analyzing an equivalent right-angle structure of an identical square span is the same in

TABLE 1.—INFLUENCE VALUES FOR VERTICAL LOADS[a]

Section	x_A	y	Δs	t	t^3	$y\frac{\Delta s^{(b)}}{I_c}$	$y^2\frac{\Delta s}{I_c}$	$V_s^{(c)}$	$\frac{\Delta x}{4.67}$	$V_s\frac{\Delta x}{4.67}$	$P_s\frac{\Delta x}{9.33}$	$\frac{M_s^{(d)}}{4.67}$	H	$P_s x_A$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
								0				992	0.437	
13	31.8	25.0	4.67	3.00	27.0	51.8	1,291	51.8	↗ 1	51.8	25.9	965	0.426	1,649
12	27.2	24.9	4.67	3.10	29.8	46.8	1,168	98.6	↗ 1	98.6	23.4	890	0.393	1,270
11	22.5	24.8	4.67	3.31	36.3	38.2	947	136.8	↗ 1	136.8	19.1	773	0.341	860
10	17.8	24.5	4.67	3.68	49.8	27.6	675	164.4	↗ 1	164.4	13.8	622	0.275	492
9	13.2	24.2	4.67	4.19	73.6	18.5	447	182.9	↗ 1	182.9	9.3	448	0.199	243
8	8.5	23.9	4.67	4.82	112.0	12.0	286	194.9	↗ 1	194.9	6.0	259	0.115	102
7	3.8	23.4	4.67	5.60	175.6	7.5	176	202.4	↗ 1	202.4	3.8	61	0.027	29
6					∞			202.4	↗ 0.60	121.4				
5	−1.1	17.9	4.54	5.30	148.9	6.5	117	208.9	↗ −0.07	−14.6	−0.2			−7
4	−0.8	13.3	4.54	4.65	100.5	7.3	97	216.2	↗ −0.07	−15.1	−0.2			−6
3	−0.5	8.8	4.54	4.00	64.0	7.5	66	223.7	↗ −0.07	−15.7	−0.3			−4
2	−0.2	4.3	4.54	3.35	37.6	6.2	26	229.9	↗ −0.07	−16.1	−0.2			−1
1	0	1.0	2.00	7.00	343	0.1	0	230.0	↗ 0	0	0			0
Σ						230.0	{5,296 / 10,592}			1,091.7	100.4			4,627

[a] See Fig. 7. [b] $y\frac{\Delta s}{I_c} = P_s$ is the "elastic load." [c] V_s is the "shear" resulting from the "elastic loads." [d] M_s is the moment resulting from the "elastic load."

TABLE 2.—DEAD-LOAD MOMENTS IN BARREL OF FRAME[a]
(IN KIPS AND FEET)

Section	x_A	x_B	y	P[b]	HF[c]	H[d]	$P\left(\frac{x_B}{L}\right)$[e]	$P\left(\frac{x_A}{L}\right)$[b]	V	Δx	$V\,\Delta x$	$P\frac{\Delta x}{2}$	M_s[f]	$H\,y$	M
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
			23.0						−30.61				0	339	−339
6′R	67.6	0.7		1.89	0.015	0.028	0.02	1.87		1.50	− 43.1	1.4			
			23.2						−28.72	↗			44	342	−298
7R	64.5	3.8		5.56	0.070	0.389	0.31	5.25		4.67	−107.9	13.0			
			23.7						−23.16	↗			165	349	−184
8R	59.8	8.5		4.91	0.156	0.765	0.61	4.30		4.67	−85.3	11.5			
			24.1						−18.25	↗			262	355	− 93
9R	55.1	13.2		4.40	0.239	1.054	0.85	3.55		4.67	− 64.6	10.3			
			24.4						−13.85	↗			337	360	− 23
10R	50.5	17.8		3.97	0.310	1.230	1.03	2.94		4.67	− 46.1	9.3			
			24.7						− 9.88	↗			392	363	+ 29
11R	45.8	22.5		3.64	0.370	1.348	1.20	2.44		4.67	− 29.0	8.5			
			24.9						− 6.24	↗			430	366	+ 64
12R	41.1	27.2		3.40	0.411	1.400	1.35	2.05		4.67	− 13.2	7.9			
			25.0						− 2.84	↗			451	368	+ 83
13R	36.5	31.8		3.28	0.432	1.416	1.53	1.75		4.67	+ 2.1	7.7			
			25.0						+0.44	↗			457	369	+ 88
13L	31.8	36.5		3.22	0.432	1.390	1.72	1.50		4.67	+ 17.1	7.5			
									+ 3.66	↗			447	368	+ 79
12L	27.2	41.1		3.25	0.411	1.340	1.95	1.30		4.67	+ 32.2	7.6			
									+ 6.91	↗			422	366	+ 56
11L	22.5	45.8		3.39	0.370	1.253	2.27	1.12		4.67	+ 48.1	7.9			
									+10.30	↗			382	363	+ 19
10L	17.8	50.5		3.61	0.310	1.119	2.67	0.94		4.67	+ 65.1	8.4			
									+13.91	↗			325	360	− 35
9L	13.2	55.1		3.94	0.239	0.942	3.18	0.76		4.67	+ 83.5	9.2			
									+17.85	↗			251	355	−104
8L	8.5	59.8		4.38	0.156	0.685	3.84	0.54		4.67	+104.0	10.2			
									+22.23	↗			157	349	−192
7L	3.8	64.5		4.93	0.070	0.345	4.65	0.28		4.67	+126.8	11.5			
									+27.16	↗			42	342	−300
6′L	0.7	67.6		1.66	0.015	0.025	1.64	0.02		1.50	+ 43.2	1.2			
									+28.82	↗			0	339	−339
Σ				59.43		14.729	28.82	30.61[a]							

[a] See Fig. 7. [b] P is the weight of a section of dead load. [c] HF is the H-influence value or factor scaled from the influence line in Fig. 7. [d] Col. 7 = Col. 5 × Col. 6. [e] Cols. 8 and 9 are equal to vertical reactions R_A and R_B, respectively. [f] M_s is the "simple beam" moment.

cross section for all four structures shown in Fig. 6. One modification is necessary, however.

It was stated previously that b, the skew width (measured parallel to the abutments), is canceled from both sides of the equations. It was also pointed

TABLE 3.—DEAD-LOAD MOMENTS AND NORMAL THRUSTS IN "LEGS"[a]
(IN KIPS AND FEET)

Section No.	x	y	(a) LEFT "LEG"				(b) RIGHT "LEG"		
			$R_A\,x_A$[b]	$H\,y$	M	N	$R_B\,x_B$	M	N
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(4)	(6)	(7)
5–6	−1.25	20.2	−36	298	−334	34	−38	−336	36
4–5	−0.95	15.6	−27	230	−257	38	−29	−259	39
3–4	−0.65	11.1	−19	164	−183	41	−20	−184	43
2–3	−0.35	6.5	−10	96	−106	43	−11	−107	44
1–2	0	2.0	0	30	− 30	45	0	− 30	46

[a] See Fig. 7. [b] $R_A\,x_A = M_s$, the "simple beam" moment.

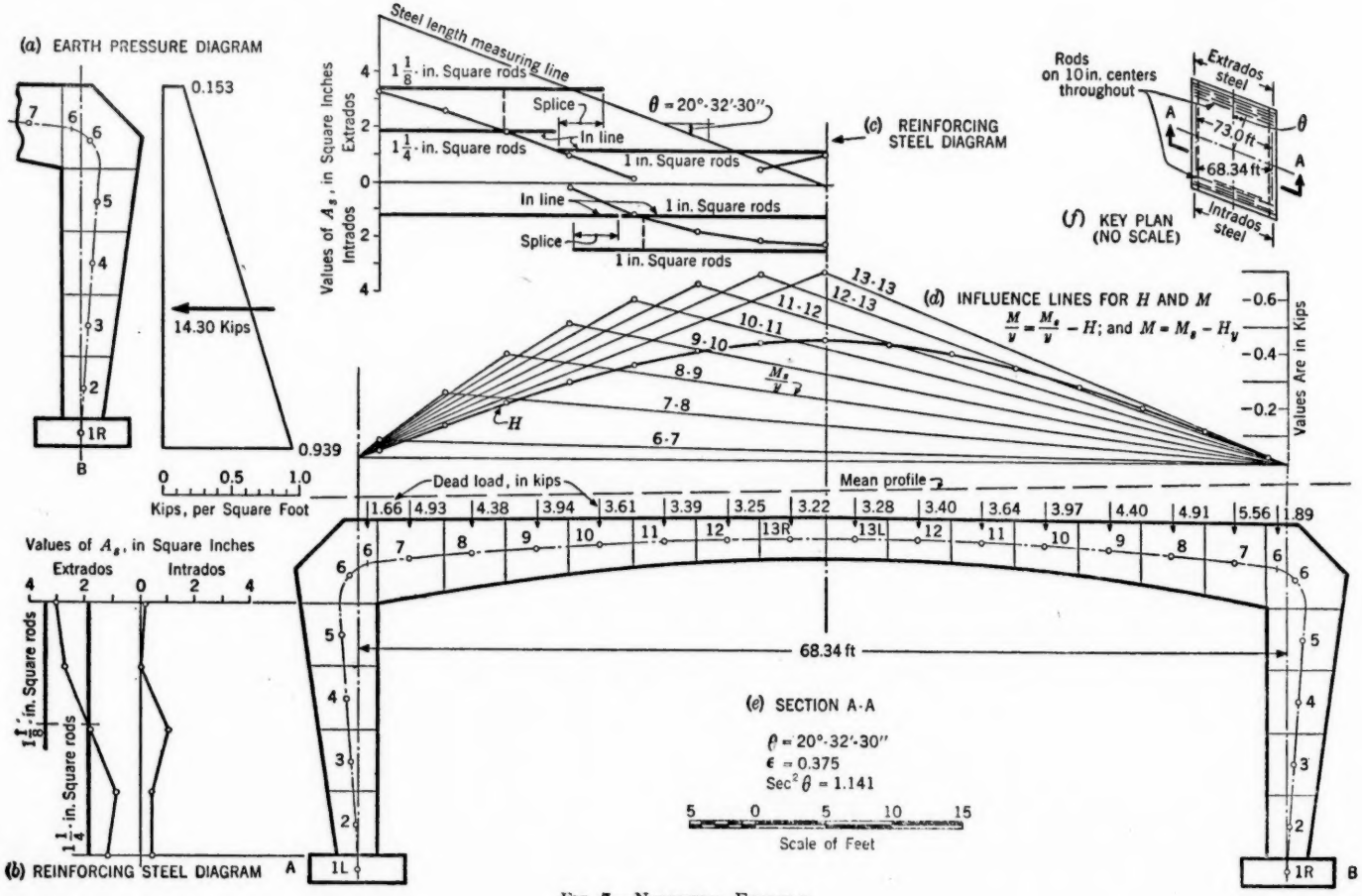

FIG. 7.—NUMERICAL EXAMPLE

TABLE 4.—TEMPERATURE CHANGE MOMENTS[a] (IN KIPS AND FEET)

Section No.	y	$H\,y^{(b)}$ (+30°)	$H\,y^{(b)}$ (−40°)
(1)	(2)	(3)	(4)
13–13	25.0	−18	24
12–13	25.0	−18	24
11–12	24.9	−18	24
10–11	24.7	−18	24
9–10	24.4	−18	23
8– 9	24.1	−18	23
7– 8	23.7	−17	23
6– 7	23.2	−17	22
5– 6	20.2	−15	19
4– 5	15.6	−11	15
3– 4	11.1	− 8	11
2– 3	6.5	− 5	6
1– 2	2.0	− 2	2
6′	23.0	−16	22

[a] See Fig. 7. [b] $H\,y$ is the final moment (since the vertical reactions are zero).

TABLE 5.—"SIMPLE BEAM" EARTH-PRESSURE MOMENTS IN "LEGS"[a] (IN KIPS AND FEET)

Section No.	x_B	y	$R\,x_B$	$Q\,y$	$-0.939\frac{y^3}{2}$	$0.005\,y^3$	$M_s^{(b)}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
5R	−1.1	17.9	2.3	256.0	−150.2	28.6	136.7
4R	−0.8	13.3	1.7	190.0	− 83.0	11.8	120.5
3R	−0.5	8.8	1.0	126.0	− 36.3	3.4	84.1
2R	−0.2	4.3	0.4	61.5	− 8.7	0.4	53.6
1R	0.0	1.0	0.0	14.3	− 0.5	0.0	13.8
5–6R	−1.25	20.2	2.6	289.0	−191.0	41.2	141.8
4–5R	−0.95	15.6	2.0	223.0	−114.0	18.9	129.9
3–4R	−0.65	11.1	1.4	159.0	− 57.7	6.9	109.6
2–3R	−0.35	6.5	0.7	93.0	− 19.8	1.4	75.3
1–2R	0.0	2.0	0.0	28.6	− 1.9	0.0	26.7

[a] See Fig. 7. [b] M_s is the "simple beam" moment.

TABLE 6.—EARTH-PRESSURE HORIZONTAL REACTIONS[a] (IN KIPS AND FEET)

Section No.	x_A	$M_s^{(b)}$	$y\frac{\Delta s}{I}$	$M_s\,y\frac{\Delta s}{I}$
(1)	(2)	(3)	(4)	(5)
1R		13.8[c]	0.1	....
2R		53.6	6.2	332
3R		84.1	7.5	630
4R		120.5	7.3	880
5R		136.7[c]	6.5	889
6R		....	....	....
7R	64.5	134.2	7.5	1,006
8R	59.8	124.3	12.0	1,490
9R	55.1	114.9	18.5	2,120
10R	50.5	105.0	27.6	2,900
11R	45.8	95.5	38.2	3,650
12R	41.1	85.5	46.8	4,000
13R	36.5	76.0	51.8	3,930
13L	31.8	66.1	51.8	3,420
12L	27.2	56.5	46.8	2,640
11L	22.5	46.8	38.2	1,790
10L	17.8	37.0	27.6	1,025
9L	13.2	27.4	18.5	507
8L	8.5	17.7	12.0	212
7L	3.8	7.9	7.5	59
6L	....	....	....	....
5L	−1.1	−2.3	6.5	−15
4L	−0.8	−1.7	7.3	−12
3L	−0.5	−1.0	7.5	− 8
2L	−0.2	−0.4	6.2	− 2
1L	0.0	0.0	0.1	0
Σ				31,444

[a] See Fig. 7. [b] M_s is the "simple beam moment." [c] See Table 5.

TABLE 7.—EARTH-PRESSURE MOMENTS[a] (IN KIPS AND FEET)

Section No.	x_A	y	$M_s^{(b)}$	$H_A\,y$	M
(1)	(2)	(3)	(4)	(5)	(6)
2– 1R		2.0	26.7[c]	5.9	20.8
3– 2R		6.5	75.3	19.3	56.0
4– 3R		11.1	109.6	33.0	76.6
5– 4R		15.6	129.9	46.3	83.6
6– 5R		20.2	141.8[c]	60.0	81.8
7– 6R	66.8	23.2	139.0	69.0	70.0
8– 7R	62.2	23.7	128.7	70.5	58.2
9– 8R	57.5	24.1	119.8	71.5	48.3
10– 9R	52.8	24.4	110.0	72.5	37.5
11–10R	48.2	24.7	100.4	73.5	26.9
12–11R	43.5	24.9	90.5	74.0	16.5
13–12R	38.8	25.0	80.8	74.2	6.6
13R–13L	34.2	25.0	71.1	74.2	− 3.1
12–13L	29.5		61.4	74.2	−12.8
11–12L	24.8		51.5	74.2	−22.7
10–11L	20.2		42.0	73.5	−31.5
9–10L	15.5		32.2	72.5	−40.3
8– 9L	10.8		22.4	71.5	−49.1
7– 8L	6.2		12.9	70.5	−57.6
6– 7L	1.5		3.1	69.0	−65.9
5– 6L			−2.6	60.0	−57.4
4– 5L			−2.0	46.3	−44.3
3– 4L			−1.4	33.0	−31.6
2– 3L			−0.7	19.3	−18.6
1– 2L			0.0	5.9	− 5.9
6′R	68.3	23.0	142.1	68.3	+73.8
6′L	0	23.0	0	68.3	−68.3

[a] See Fig. 7. [b] M_s is the "simple beam moment." [c] See Table 5.

out that this was merely a numerical convenience and not a necessity. For Figs. 6(*b*), 6(*c*), and 6(*d*), *b* cannot be canceled out. However, the equations can still be divided by the width of some section (or by some arbitrary number, for that matter) to reduce the size of the numbers handled. If the former procedure is followed, the value of *I* used will, in effect, be divided by a magnitude b_o and the resulting total stresses instead of being divided by *b* will now be divided by b_o. However, to obtain the total stress at any section, the stress computed will have to be multiplied by b_o/b and the result can then be treated as in the analysis of a skew frame.

TABLE 8.—SUMMARY OF BASIC STRESSES;

Line	Loading	(*a*) BARREL OF THE FRAME ($\phi = 0°$; $\theta' = \theta$; AND $\sec^2\theta'$											
		6–7L		7–8L		8–9L		9–10L, R		10–11L, R		11–12R	
		M	*N*	*M*	*N*	*M*	*N*	*M*	*N*	*M*	*N*	*M*	*N*
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
1	Dead load (DL)	−300	15	−192	15	−104	15	− 35L − 23R	15 15	+ 29R + 19L	15 15	+ 64	15
	Earth Pressure (EP)—												
2	Near side	+ 70	3	+ 58	3	+ 48	3	+ 38	3	+ 27	3	+ 17	3
3	Far side	− 66	3	− 58	3	− 49	3	− 40	3	− 32	3	− 23	3
	Live Load (LL)—												
4	$+M$							+ 19	1	+ 22	1	+ 23	2
5	$-M$	− 45	2	− 33	2	− 23	2	− 16	2	− 10	1		
	Temperature Change ($\Delta\tau$)												
6	$+\overline{30°}$	− 17	1	− 17	1	− 18	1	− 18	1	− 18	1	− 18	1
7	$-\overline{40°}$	+ 22	−1	+ 23	−1	+ 23	−1	+ 23	−1	+ 24	−1	+ 24	−1
8	$\overline{DL}+2\overline{EP}$ (optional)	−300	15	−192	15	−105	21	− 37	21	+ 29	15	+ 64	15
9[a]	Line $8+\overline{LL}$	−345	17	−225	17	−128	23	− 53	23	+ 51	16	+ 87	17
10[a]	$\overline{DL}+\overline{LL}+1\ \overline{EP}+\Delta\tau$	−428	21	−300	21	−194	21	−109L + 57R	21 18	−102R − 41L	18 20	+128	19

[a] Lines 9 and 10 are cases

For the design of the sections of the bridges shown in Figs. 6(*b*) and 6(*c*), the two faces are designed separately, each for its own particular skew angle. The change in the reinforcing steel, from the direction parallel to one face to the direction parallel to the other, can be made either by "fanning" the rods near the middle of the structure or by intersecting the two sets of steel at some convenient section, according to the practice of the individual designer.

In the case of the curved bridge shown in Fig. 6(*d*), the skew angle varies at every point; but this is no great hardship. As the radii of the curves will be fairly large (for practical cases), the variation will not be very rapid and only a few skew angles need to be used in the design of the sections. Even if the maximum angle is used, no great loss in economy will result. If the two faces of the bridge are concentric arcs, the reinforcing steel can be curved and placed parallel to the faces of the structure. If the faces are not parallel, modifications similar to those suggested for splayed bridges can be used by the designer. The practice of curving the steel is not new. Bridges with curved

soffits require steel bent in a vertical plane; curving in a horizontal plane should present no difficulty.

Design of a Single-Span, Skew, Rigid Frame; Numerical Example

The data for the design of an actual rigid frame are given in Fig. 7. Hinged ends are assumed, and a correction is made, subsequently (in accordance with current practice), for the fixity of the footings. Therefore, the right-frame, variable section design is a typical example of standard office procedure, and Tables 1 to 9 need little further explanation. "Elastic loads" are used to

See Fig. 7 (in Kips and Feet)

= 1.141)		(b) "Legs" ($\phi = 90°$; $\theta' = 0°$; and $\sec^2\theta' = 1$)														
12–13R		13–13		6′R		5–6R		4–5R		3–4R		2–3R		1–2R		Line
M	N	M	N	M	N	M	N	M	N	M	N	M	N	M	N	
(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)	(29)	(30)	
+ 83	15	+ 88	15	−399	33	−336	36	−259	39	−184	43	−107	44	−30	46	1
+ 7	3	− 3	3	+ 74	−2	+ 82	−2	− 84	−2	+ 77	−2	+ 56	−2	+21	−2	2
− 13	3	− 3	3	− 68	2	− 57	2	− 44	2	− 32	2	− 19	2	− 6	2	3
+ 23	2	+ 23	2													4
				− 48	6	− 43	6	− 34	6	− 24	6	− 14	6	− 4	6	5
− 18	1	− 18	1	− 16	0	− 15	0	− 11	0	− 8	0	− 5	0	− 2	0	6
+ 24	−1	+ 24	−1	+ 22	0	+ 19	0	+ 15	0	+ 11	0	+ 6	0	+ 2	0	7
+ 83	15	+ 88	15	−399	33	−336	36	−259	39	−184	43	−107	44	−30	46	8
+106	17	+111	17	−387	39	−379	42	−293	35	−208	49	−121	50	−34	52	9[a]
+137	19	+135	16	−471	41	−451	44	−348	47	−248	51	−145	52	−42	54	10[a]

I and II, respectively.

obtain the constants for computing influence lines. The temperature stresses are modified as explained in the text. Table 9 demonstrates the final step, the design of the sections, which (except for the correction required by the modified skew angle θ') is little different from a right-frame design.

In Table 1, the data in Cols. 2 and 5 are scaled from the layout in Fig. 7, (x_A and x_B being the abscissas measured from points A and B, respectively, and y being the ordinate measured from line AB), from which the values in Cols. 6, 7, and 8 follow directly by slide-rule computation. The values for $\left(y\frac{\Delta s}{I}\right)$ in Col. 7 can be considered the "elastic loads," P_e, in order to compute the deflections $\Sigma M_s\, y\frac{\Delta s}{I}$ for substitution in the formula:

$$H = \frac{\Sigma M\, y\frac{\Delta s}{I}}{\Sigma y^2\frac{\Delta s}{I}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (24)$$

which results in the influence line for vertical loads. In Col. 9, V_e is the vertical "elastic shear" resulting from the "elastic loads," P_e. Since the structure is symmetrical, the sum of the loads for half the bridge is the reaction—that is, 230.0 from Col. 7. In the elastic load method, the "elastic moment" at any point is the deflection at that point. Cols. 9 to 13, inclusive, show the computation of these moments by shear and moment increments. The hori-

TABLE 9.—DESIGN

Description	Case I
Concrete stress, f_c, in pounds per square inch	800
Steel stress, f_s, in pounds per square inch	20,000
$K_1 = \left(K = \frac{M_s}{b\,d^2}\right)$ for a balanced section, in pounds per square inch	116
$a = \frac{f_s j}{12}$, which for a balanced section, is	1.49

Section No.	Case	$M \sec^2 \theta'$ (K')	$N \sec^2 \theta'$ (K)	t (in.)	d (in.)	r (ft)[a]	$N' r$ (kip-feet)	M_s (kip-feet)[b]	K (lb per sq in.)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
13–13	I	127.0	17.1	36	33.5	1.29	22.1	149	133
13–13	II	154.2	19.4	36	33.5	1.29	25.0	179	160
12–13	I	121.0	19.4	37	34.5	1.33	25.8	147	124
12–13	II	156.5	21.7	37	34.5	1.33	28.9	185	156
11–12	II	141.8	21.7	38	35.5	1.37	29.8	172	137
10–11	II	116.6	20.6	42	39.5	1.54	31.8	148	96
10–11	II	−46.7	22.8	42	39.5	1.54	35.1	82	53
9–10	II	65.0	20.6	47	44.5	1.75	36.0	101	51
9–10	II	−124.5	24.0	47	44.5	1.75	42.0	167	84
8– 9	II	−221.5	24.0	54	51.5	2.04	49.0	270	102
7– 8	II	−342	24.0	62	59.5	2.37	56.9	399	113
6– 7	I	−396	19.3	72	69.5	2.79	53.8	450	93
6– 7	II	−487	24.0	72	69.5	2.79	66.9	554	115
5– 6	I	−379	42	67	64.5	2.58	108.7	488	118
5– 6	II	−451	44	67	64.5	2.58	113.6	565	136
4– 5	I	−293	35	59	56.5	2.25	78.8	372	117
3– 4	I	−208	49	51	48.5	1.92	94.0	302	129
2– 3	I	−121	50	43	40.5	1.58	79.0	200	122
1– 2	I	− 34	52	36	33.5	1.29	67.0	101	90
1– 2	I	85	52	36	33.5	1.29	67.0	152	136

[a] $r = \frac{t}{2} - 0.21$. [b] $M_s = M' + N' r$. [c] $M_2 = M_s \left(\frac{K - K_1}{K}\right)$.

zontal reactions, H, in Col. 14 are obtained by dividing the values of M_e from Col. 13 by $\Sigma y^2 \frac{\Delta s}{I} = 10{,}592$ from Col. 8.

By symmetry, the "elastic moment" at the center line equals $\Sigma(P_e x_A)$ which is the value of the deflection $\Sigma\left(M\,y\,\frac{\Delta s}{I}\right)$ at this point, and substitut-

ing the summation from Col. 15 into the formula for the horizontal reaction: $H = \frac{4{,}630}{10{,}590} = 0.437$, which is the same as the corresponding value in Col. 14. Thus Col. 15 serves as a check on the numerical work leading up to and including Col. 14.

In Table 2, the horizontal reaction for dead load is obtained by multiplying

OF SECTIONS

Case II
1,000
25,000
145
186

$n = 12$
$k = 0.324$
$j = 0.892$
$p = 0.0065$

M_2 (kip-feet)[c]	$C_2 = T_2$ (kips)[d]	Kips per Square Inch		Square Inches			N' f_s (sq in.)	A_s (sq in.)	Section No.
		f_c	f'_s	A'_s[e]	A_{s1}	A_{s2}[e]			
(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	
19.0	7.4	20	6.7	1.10	2.61	0.37	0.85	2.13	13–13
16.8	6.5		8.4		2.61	0.26	0.78	2.09	13–13
9.5	3.6	20	6.8	0.53	2.69	0.18	0.87		12–13
13.1	4.9	25	8.5	0.58	2.69	0.20	0.87	2.02	12–13
		25			2.59		0.87	1.72	11–12
		25			1.97		0.82	1.15	10–11
		25			1.08		0.91	0.17	10–11
		25			1.22		0.82	0.40	9–10
		25			1.97		0.96	1.01	9–10
		25			2.77		0.96	1.81	8– 9
		25			3.56		0.96	2.60	7– 8
		20			4.41		0.77	3.64	6– 7
		25			4.21		0.96	3.25	6– 7
8.3	1.6	20	7.7	0.21	5.03	0.08	2.10	3.01	5– 6
		25			4.70		1.76	2.94	5– 6
3.2	0.7	20	7.6	0.09	4.40	0.04	1.75	2.69	4– 5
30.4	7.9	20	7.4	1.07	3.78	0.40	2.45	1.73	3– 4
9.8	3.1	20	7.1	0.44	3.16	0.16	2.50	0.82	2– 3
		20			1.59		2.60	Nominal	1– 2
22.4	8.7	20	6.7	1.13	2.62	0.44	2.60	0.46	1– 2

[d] $C_2 = T_2 = \frac{M_2}{2\,r}$. [e] A'_s (Col. 15) $= \frac{C_2}{f'_s}$; and A_{s2} (Col. 17) $= \frac{T_2}{f_s}$.

the weight of a section by the influence line ordinate ($H\,F$) scaled from the diagram in Fig. 7. The computation of the "simple beam moment" and the negative moment $H\,y$ is obvious. The former moment is computed by shear and moment increments. The value of H computed in Table 2 is also used in Table 3.

To obtain the horizontal reactions for temperature change, use is made of the relationship:

$$H = H'' \sec^2 \theta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (25)$$

in which H is the "basic" horizontal reaction in a skew frame and H'' is the corresponding reaction in a right frame. Since

$$H'' = \frac{c\,t\,L}{\Sigma y^2 \frac{\Delta s}{E\,I}} = \frac{E\,c\,t\,L}{\Sigma y^2 \frac{\Delta s}{I}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (26)$$

it is easily computed. Using kip and foot units, $E\,c\,t\,L = 432{,}000 \times 0.0000065 \times 100° \times 68.3 = 19{,}200$; $\sec^2 \theta = 1.141$; and $\Sigma \left(y^2 \frac{\Delta s}{I_c} \right) = 10{,}590$.

Assuming that I_s, the moment of inertia of the steel, equals 15% of I_c, the moment of inertia of the concrete; and, since $I = I_c + I_s = 1.15\,I_c$, $H = \frac{19{,}200 \times 1.15 \times 1.141}{10{,}590} = 2.38$ kips for + 100° F. Therefore, $H = 0.715$ kips for + 30° F; and H = − 0.951 kips for − 40° F. With these values for H, Table 4 is readily computed.

To obtain the loads Q and "simple beam" reactions R_A and R_B for earth pressure acting on one side of the structure (Fig. 7), moments are taken about point B:

$0.153 \times 13.10 =$	2.00 kips $\times$ 17.47 =	34.9 kip-ft
$0.939 \times 13.10 =$	12.30 kips $\times$ 8.73 =	107.2 kip-ft
$Q =$	14.30 kips	$M_B =$	142.1 kip-ft

The vertical reaction $R_A = \frac{142.1}{68.3} = 2.08$ kips $= -\,R_B$, from which Table 5 follows directly. This table is a preparatory computation and simplifies the work involved in preparing subsequent tables.

To compute the horizontal reactions for earth pressure acting on one side of the structure, use is again made of Eq. 24. The numerator of this equation is computed in Table 6 using some of the values from Table 5, from which $H_A = \frac{31{,}400}{10{,}590} = 2.97$ kips. Since H is now known, the moments due to earth pressure acting on one side of the structure are computed in Table 7. The remaining values computed in Table 5 are brought over to this table.

Table 8 shows a summary of the "basic moments and thrusts," M and N, respectively, for the loading conditions already computed. The live-load moments and thrusts are also given. These are obtained directly from the influence lines shown in Fig. 7 and need no further explanation.

The assumption of hinged footings has been found to be slightly on the side of safety at the crown and immaterial at the springing line. The only sections that need further investigation are at the top of footings, as may be seen from Fig. 8. However, even these sections have sufficient reinforcing steel in the form of dowels to resist the positive moment that may be induced by the fixity of the footing. A close estimate of the steel required for this

moment can be made by assuming an average value for the moment at the bottom of the fixed footing to be about -40% of the moment at point 6′ (see Fig. 7): $M_{1-2} = 0.4\,M_{6'} - 1.4\,M_{6'} \times \frac{3}{23} = 0.22\,M_{6'} = 0.22 \times 387$ kip-ft. Therefore $M_{1-2} = 85$ kip-ft, the thrust remaining practically the same as for the hinged condition.

Table 9 shows the "design of the sections" and except for the fact that the "basic moments and thrusts" are multiplied by $\sec^2 \theta'$ (the values of which are given in the table), this computation is otherwise exactly the same as that for a right-angle structure.

Fig. 7 shows a typical main-steel pattern for a rigid frame. The spacing given is measured normal to the steel that is parallel to the fascia. To find the true length of the bars, they are projected onto and scaled along the "steel length measuring line."

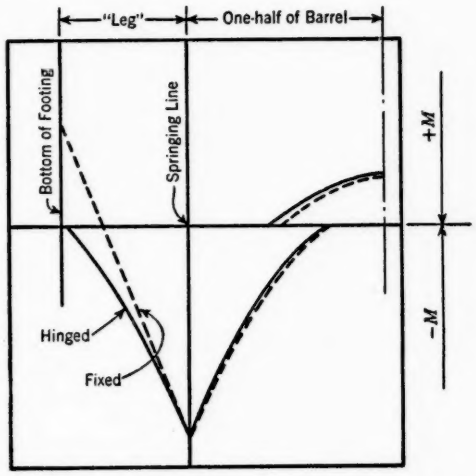

FIG. 8.—RELAITVE MAXIMUM MOMENTS FOR HINGED AND FIXED FOOTINGS

CONCLUSION

A skew frame can be designed with little more labor than that required for a similar right-angle frame. For clarity, the salient features of a design are given in a numerical example (under the heading, "Design of a Single-Span, Skew, Rigid Frame; Numerical Example") based on the following considerations:

1. With more than sufficient accuracy for practical design purposes, the "basic stresses" and the "skew stresses" (total) may be separated and treated as separate elastic systems.

2. The basic stresses, except for temperature and shrinkage, are very closely equal to those in a similar right-angle structure with the same square span and with the same elastic properties. For temperature and shrinkage, the basic stresses obtained from the same right-frame analysis are multiplied by the square of the secant of the skew angle.

3. For the fascia section, only the basic stresses need to be known. The resultant stresses must be parallel to the boundary (free surface or face of bridge) from which it follows that the bending moment and thrust per unit width are multiplied by the square of the secant of the modified skew angle (θ'). The sections are then designed for these increased stresses in the same way as for a right-angle structure.

4. The fascia section is adequate for the entire width of structure from the points of view of both safety and economy.

Regardless of other considerations, the stresses at the fascia must have a resultant that is parallel to the boundary. This requirement follows entirely from considerations of equilibrium and is quite independent of any and all skew-arch theories. The stresses at the fascia being more certain, therefore, sound judgment should dictate the use of the fascia design, which is on the side of safety, for the entire width of structure. Without good experimental data, refinements do not seem to be justified.

Attention is also called to the fact that the total stresses—however computed—must first be reduced to stresses per unit width before the sections can be designed. In 1930, H. M. Westergaard, M. ASCE, described an analogous case of such treatment, including the finding of maximum moments per unit width by an extension of the principles involved in the ellipse of stress.[8]

Transverse Reinforcement

The question of transverse reinforcement is as much a problem in the design of right-angle frames as it is in the design of skew frames. The latter, as has been shown herein, needs no special treatment; in either case the reinforcement is required for transverse load and stress distribution. As far as dead load is concerned, only nominal reinforcement is required since any unbalance is not likely to be very large even for a large skew and a steep top grade. However, reinforcement for the distribution of unbalanced live load—unbalance in the transverse direction—is necessary.

It is suggested that a fair estimate of the transverse reinforcement can be obtained by considering the horizontal member of the frame as a simply supported slab between points of contraflexure. Using the distance between the points of contraflexure as the span and the average moment of inertia between these two points, the transverse steel necessary to distribute the concentrated live load may be obtained in the same manner as for any ordinary roadway bridge slab. Mr. Westergaard's equations[8] or any of the numerous approximations give reasonable results.

Theoretically, since no live load is applied directly to the "legs," no transverse reinforcement is necessary. However, the same transverse reinforcement used in the barrel is generally also used in the "legs." Since the percentage of steel computed as suggested in the previous paragraph is rather light, there is no loss in using the same steel in the "legs." In fact, the steel required is no greater than the quantity that would normally be used for temperature and shrinkage reinforcement.

[8] "Computation of Stresses in Bridge Slabs Due to Wheel Loads," by H. M. Westergaard, *Public Roads*, March, 1930, p. 1.

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